Exercise 39

(a) Use the definition of a derivative to find $f'(2)$, where $f(x) = x^3 - 2x$.

(b) Find an equation of the tangent line to the curve $y = x^3 - 2x$ at the point $(2, 4)$.

(c) Illustrate part (b) by graphing the curve and the tangent line on the same screen.

Solution

To determine $f'(2)$, use the definition of the derivative to determine $f'(x)$ and then set $x = 2$ in the result.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{[(x+h)^3 - 2(x+h)] - (x^3 - 2x)}{h} \\ &= \lim_{h\to 0} \frac{[(x^3 + 3x^2h + 3xh^2 + h^3) - 2x - 2h] - x^3 + 2x}{h} \\ &= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 - 2h}{h} \\ &= \lim_{h\to 0} (3x^2 + 3xh + h^2 - 2) \\ &= 3x^2 - 2 \end{aligned}$$

Therefore,

$$f'(2) = 3(2)^2 - 2 = 10.$$

Use the point-slope formula to obtain the equation of the line with this slope that passes through the point $(2, 4)$.

$$\begin{aligned} y - 4 &= 10(x - 2) \\ y - 4 &= 10x - 20 \\ y &= 10x - 16 \end{aligned}$$

Plot both the curve and the tangent line on the same graph. Notice that the line touches the curve at $x = 2$.